Applications

1. **a.** Ben claims that 12 is a factor of 24. How can you check whether he is correct?

   b. How would you test whether 7 is a factor of 291?

2. **a.** What factor is paired with 6 to give 24?

   b. What factor is paired with 5 to give 45?

   c. What factor is paired with 3 to give 24?

   d. What factor is paired with 6 to give 54?

3. **a.** Which of the numbers 19, 21, 23, and 25 has the most factors?

   b. Find a number greater than 25 that has more factors than the numbers in part (a).

   c. Find a number smaller than 19 that has more factors than the numbers in part (a).

4. **a.** The calculator screen below shows the result of dividing 84 by 14.

   84/14

   6

   What does the answer tell you about 14 and 84?

   b. The calculator screen below shows the result of dividing 84 by 15.

   84/15

   5.6

   What does the answer tell you about 15 and 84?

5. **a.** Is 6 a divisor of 18? Why?

b. Is 18 a divisor of 6? Why?

6. Which of these numbers are divisors of 64?

2 6 8 12 16

7. Sam knows that 3 is a factor of 24. This means that $3 \times \square = 24$. Another way to write this is $3 \times n = 24$. For each statement, find a value of n that makes the statement true.

a. $3 \times n = 24$

b. $5 \times n = 60$

c. $12 \times n = 144$

d. $160 = 8 \times n$

e. $2 \times 3 \times n \times n = 54$

8. In the following Factor Game, some of the scores are missing on the tally sheet.

Cathy	Keiko
24	36
21	?
?	9
?	30

a. In Round 1, Cathy chose 24. Keiko scored 36 points. Make a game board. Mark Cathy's choice and Keiko's factors.

b. In Round 2, it is Keiko's turn. Cathy scored 21 points. What number did Keiko pick?

c. In Round 3, it is Cathy's turn. Keiko scored 9 points. What number might Cathy have picked?

d. In Round 4, it is Keiko's turn. Keiko chose 30. What did Cathy score?

e. In Round 5, it is Cathy's turn. What is Cathy's best move? Explain your choice.

9. Lareina understands factors. Sometimes she has trouble finding all of the factors of a number. What advice would you give Lareina to help her find all of the factors of a number? Demonstrate by finding all of the factors of 110.

10. **a.** Find two numbers that have 2, 3, and 5 as factors. What other factors do the two numbers have in common?

b. Find three numbers that have 2, 4, and 8 as factors. What do these numbers have in common?

11. **a.** Suppose you circle a prime number in the Factor Game. Your opponent receives at most one point. Explain why. Use examples.

b. Suppose you circle a composite number in the Factor Game. Your opponent might receive more points than you. Explain why. Give some examples.

12. The expression $3 \times n$ can be written without the multiplication sign as $3n$. For each expression in parts (a)–(c),

- Use $n = 1, 2, 3, \ldots, 10$ to evaluate the expression.
- Describe the set of ten numbers.
- Suppose the sequence continues for values of n greater than 10. Do the numbers 12, 21, 30, or 210 appear in the sequence?

a. $3n$ **b.** $7n$ **c.** $6n$

13. The Factor Game can be played on a 49 board that includes the whole numbers from 1 to 49.

The Factor Game

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

a. Extend your table for analyzing first moves on a 30 board to include all of the numbers on a 49 board.

b. What new primes can you find?

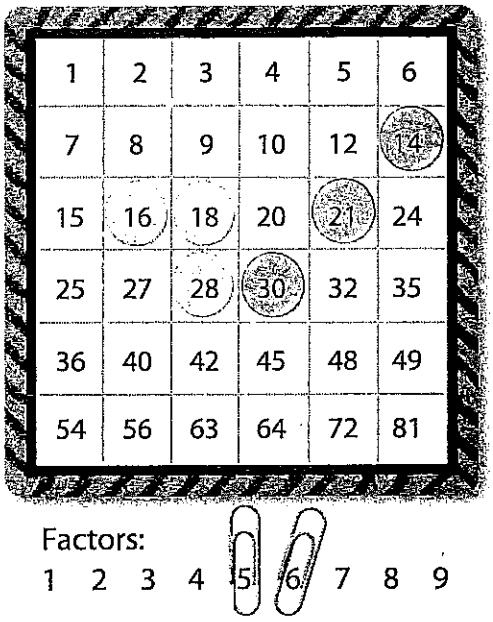
c. What new square numbers can you find?

d. What is the best first move on a 49 board? Why?

e. What is the worst first move on a 49 board, other than 1? Why?

14. Dewayne and Todd are playing the Product Game. Dewayne's markers are on 16, 18, and 28. Todd's markers are on 14, 21, and 30. The paper clips are on 5 and 6. It is Dewayne's turn to move a paper clip.

The Product Game



a. List the moves Dewayne can make.

b. Which move(s) would give Dewayne three markers in a row?

c. Which move(s) would allow him to block Todd?

d. Which move do you think Dewayne should make? Explain.

15. a. Suppose that one paper clip on the Product Game board is on 3. What products can you make by moving the other paper clip?

b. List five multiples of 3 that are not on the game board.

c. How many multiples of 3 are there on the game board?

16. a. Davis just marked 18 on a Product Game board. On which factors might the paper clips be placed? List all of the possibilities.

b. Are there other factors of 18?

17. Determine whether each number can be made in more than one way in the Product Game. Say whether the number is prime or composite.

a. 36 **b.** 5 **c.** 7 **d.** 9

18. Salvador said that the Product Game could be called the Multiple Game. Do you agree? Why or why not?

19. a. On the Product Game board, which number is both a prime number and an even number?

b. Is there another even prime number? Explain.

20. a. What three factors were used to create this Product Game board?

4	6	9
14	?	49

Factors: ___ ___ ___

b. What product is missing from the grid?

21. a. What four factors were used to create this Product Game board?

9	15	18	
21	?	30	35
	36	42	49

Factors: ___ ___ ___ ___

b. What product is missing from the grid?

For Exercises 22–27, use the dimensions of each rectangle that can be made from the given number of tiles to list all of the factor pairs for each number.

22. 24 **23.** 32 **24.** 48

25. 45 **26.** 60 **27.** 72

28. a. What type of number has exactly two factors? Give examples.

b. What type of number has an odd number of factors? Give examples.

c. Are there any prime numbers that are also square numbers? Give an example or explain why not.

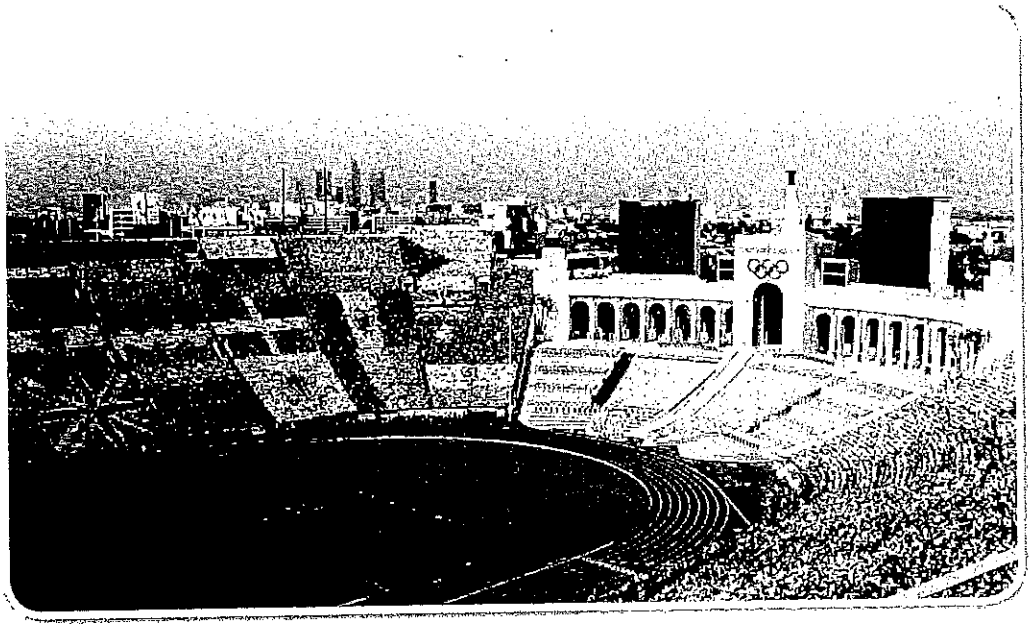
29. How many rectangles can you build with a prime number of square tiles?

30. **Multiple Choice** Which of these numbers is a square number?

A. 128 **B.** 225 **C.** 360 **D.** 399

31. Luke has chosen a mystery number. His number is greater than 12 and less than 40. It has exactly three factors. What might his number be? Use the display of rectangles from Problem 1.4 to find Luke's number. Also, think about what the displays for the numbers 31 to 40 would look like.

32. The Olympic photograph below inspired a school pep club to design card displays for football games. Each display uses 100 square cards. At a game, groups of 100 volunteers will hold up the cards to form complete pictures. They are most effective when the volunteers sit in a rectangular arrangement. What rectangular seating arrangements are possible? Which arrangements would you choose? Why?



33. A school band has 64 members. The band marches in the shape of a rectangle.

a. What rectangles can the band director make by arranging the band members?

b. Which of these arrangements is most appealing to you? Why?

Connections

34. As part of the carnival, the school will hold a Factor Game marathon. It takes Archie and Kel an average of 12 minutes to finish one game. About how many games will they finish if they play nonstop from 9:00 A.M. to 2:30 P.M.?

35. **Multiple Choice** Carlos read a book for language arts class. He finished the book on Friday. On Monday he read 27 pages. On Tuesday he read 31 pages. On Wednesday he read 28 pages. On Thursday and Friday he read the same number of pages each day. The book has 144 pages. How many pages did he read on Thursday?

A. 28 **B.** 29 **C.** 31 **D.** 58

36. The variable n represents a whole number.

a. For what values of n will the sum $n + 3$ be less than 50?

b. For what values of n will the product $3n$ be less than 50?

37. Long ago, people decided to divide the day into units called hours. They chose 24 as the number of hours in one day. Why is 24 a more convenient choice than 23 or 25?

38. In developing the ways of calculating time, astronomers divided an hour into 60 minutes. Why is 60 a better choice than 59 or 61?

39. **a.** Ms. Diaz wants to divide her class of 30 students into 10 groups. The groups do not need to be of equal size. What are some of her choices?

b. Ms. Diaz wants to divide her class of 30 students into equal-sized groups. What are her choices?

c. How is the thinking you did in part (a) different from the thinking you did in part (b)?

40. Allie's aunt has saved $10,000 in $20 bills. She spends one $20 bill every day. How many days will it take her to run out of bills?

Extensions

41. Jocelyn and Moesha decide to play the Factor Game on a 100-board. A 100-board includes the whole numbers from 1 to 100.

a. What will Jocelyn score if Moesha chooses 100 as her first move?

b. What will Jocelyn score if Moesha chooses 99 as her first move?

c. What is the best first move on a 100-board?

42. What number am I?

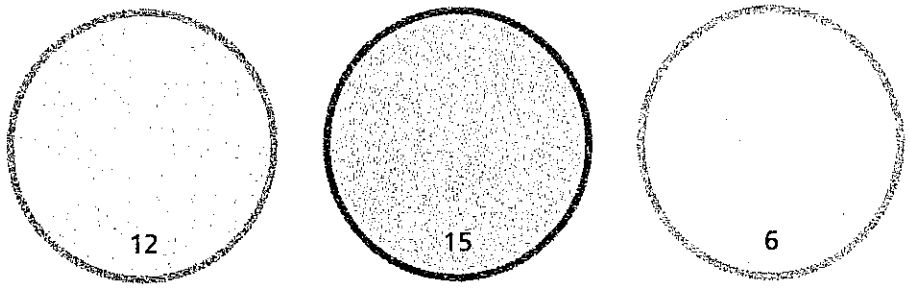
Clue 1 When you divide me by 5, the remainder is 4.

Clue 2 I have two digits. Both digits are odd.

Clue 3 The sum of my digits is 10.

43. The sum of the proper factors of a number may be *greater than*, *less than*, or *equal to* the number. Mathematicians use this idea to classify numbers as *abundant*, *deficient*, or *perfect*. Each whole number greater than 1 falls into one of these three categories.

a. Draw and label three circles as shown below. The numbers 12, 15, and 6 have been placed in the appropriate circles.



Use your factor list to determine what each label means. Then, write each whole number from 2 to 30 in the correct circle.

b. Are the labels appropriate? Why or why not?

c. In which circle does 36 belong?

d. In which circle does 55 belong?

44. **a.** Suppose you choose 16 as a first move in the 49 Factor Game. How many points does your opponent get? How does your opponent's score for this turn compare to yours?

b. Suppose you choose 4 as a first move. How many points does your opponent get? How does your opponent's score for this turn compare to yours?

c. Find another number on the 49 Factor Game Board that has the same pattern of scoring as 4 and 16. These numbers are called *near-perfect* numbers. Why do you think this name fits?

d. Examine the factor lists for the near-perfect numbers. Use this information to find two more near-perfect numbers.

45. Find three numbers you can multiply together to get 300.

46. **a.** This is the complete list of the proper factors of a certain number.

1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 49, 84, 98, 147, 196, 294

What is the number?

b. List each of the factor pairs for the number.

c. How is the list of factor pairs related to the rectangles that can be made for the factor pairs?

Did You Know?

Is there a largest perfect number? Mathematicians have been trying for hundreds of years to answer this question. After 6 and 28, the next perfect number is 496.